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COMMENTS AND REPLIES

Comment on 'Single-mode excited entangled coherent states'

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Abstract
In (Xu and Kuang 2006 *J. Phys. A: Math. Gen.* **39** L191), the authors claim that, for single-mode excited entangled coherent states $|\Psi_{\pm}(\alpha, m)\rangle$, 'the photon excitations lead to the decrease of the concurrence in the strong field regime of $|\alpha|^2$ and the concurrence tends to zero when $|\alpha|^2 \to \infty$'. This is wrong.

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In the recent paper [1], single-mode excited entangled coherent states (SMEECSs) $|\Psi_{\pm}(\alpha, m)\rangle$ are introduced and their entanglement characteristics and the influence of photon excitations on quantum entanglement are also investigated. They claim that 'the photon excitations lead to the decrease of the concurrence in the strong field regime of $|\alpha|^2$ and the concurrence tends to zero when $|\alpha|^2 \to \infty$'. Unfortunately, however, this conclusion is wrong.

First, we recall the entangled coherent states (ECS) [2]

$$|\Psi_{\pm}(\alpha, 0)\rangle = N_{\pm}(\alpha, 0)(|\alpha, \alpha\rangle \pm |-\alpha, -\alpha\rangle), \tag{1}$$

where $|\alpha, \alpha\rangle \equiv |\alpha\rangle_a \otimes |\alpha\rangle_b$ with $|\alpha\rangle_a$ and $|\alpha\rangle_b$ being the usual coherent states in a and b modes, respectively, and

$$(N_{\pm}(\alpha, 0))^{-2} = 2[1 \pm \exp(-4|\alpha|^2)] \tag{2}$$

is the normalization constants.

The SMEECSs are obtained through actions of a creation operator of a single-mode optical field on the ECSs, which are expressed as

$$|\Psi_{\pm}(\alpha, m)\rangle = N_{\pm}(\alpha, m)a^{\dagger m}(|\alpha, \alpha\rangle \pm |-\alpha, -\alpha\rangle), \tag{3}$$

where without any loss of generality we consider m-photon excitations of the mode a in the ECS and $N_{\pm}(\alpha, m)$ represents the normalization factor. Using the identity of operator [3]

$$a^n a^{\dagger m} = (-\mathrm{i})^{n+m} : H_{m,n}(\mathrm{i}a^{\dagger}, \mathrm{i}a) : \tag{4}$$

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where the symbol : : represents the normal ordering for Bosonic operators $(a^\dagger, a)$, and $H_{m,n}(\eta, \eta^*)$ is the two-variable Hermite polynomial [4],

$$H_{m,n}(\eta, \eta^*) = \sum_{l=0}^{\min(m,n)} \frac{(-1)^l n! m!}{l!(m-l)!(n-l)!} \eta^{m-l} \eta^{*n-l}, \tag{5}$$

we can easily obtain

$$\langle\alpha| a^m a^{\dagger m} |\alpha\rangle = m! L_m(-|\alpha|^2), \qquad \langle\alpha| a^m a^{\dagger m} |-\alpha\rangle = m!\, \mathrm{e}^{-2|\alpha|^2} L_m(|\alpha|^2), \tag{6}$$

and directly calculate the normalization factor

$$[N_\pm(\alpha, m)]^{-2} = 2m![L_m(-|\alpha|^2) \pm \mathrm{e}^{-4|\alpha|^2} L_m(|\alpha|^2)], \tag{7}$$

where $L_m(x)$ is the m-order Laguerre polynomial defined by [5]

$$L_m(x) = \sum_{l=0}^{m} \frac{(-1)^l m! x^l}{(l!)^2 (m-l)!}. \tag{8}$$

It is quite clear that when $m = 0$, equation (3) reduces to the usual ECSs in equation (1). Equation (7) is valid for any integer m (including the case of $m = 0$) which is different from equation (4) of [1].

Next, we calculate the concurrence for the SMEECSs. Noting that the photon-added coherent states (PACSs) $|\alpha, m\rangle$ are defined by [5]

$$|\alpha, m\rangle = \frac{a^{\dagger m}|\alpha\rangle}{\sqrt{m! L_m(-|\alpha|^2)}}, \tag{9}$$

thus the SMEECSs $|\Psi_\pm(\alpha, m)\rangle$ in terms of the PACSs can be rewritten as

$$|\Psi_\pm(\alpha, m)\rangle = M_\pm(\alpha, m)(|\alpha, m\rangle \otimes |\alpha\rangle \pm |-\alpha, m\rangle \otimes |-\alpha\rangle), \tag{10}$$

where the normalization constant $M_\pm(\alpha, m)$ is determined by

$$[M_\pm(\alpha, m)]^2 = \frac{L_m(-|\alpha|^2)}{2[L_m(-|\alpha|^2) \pm \mathrm{e}^{-4|\alpha|^2} L_m(|\alpha|^2)]}. \tag{11}$$

Following the approach of [6] and considering equations (6) and (9), for the SMEECSs $|\Psi_\pm(\alpha, m)\rangle$, the concurrence can be calculated as

$$C_\pm(\alpha, m) = \frac{\sqrt{\left(1 - p_1^2\right)\left(1 - p_2^2\right)}}{1 \pm p_1 p_2}, \tag{12}$$

where

$$p_1 = \langle\alpha, m| - \alpha, m\rangle = \frac{\exp(-2|\alpha|^2) L_m(|\alpha|^2)}{L_m(-|\alpha|^2)}, \tag{13}$$

and

$$p_2 = \langle\alpha| - \alpha\rangle = \exp(-2|\alpha|^2). \tag{14}$$

Then submitting equations (13) and (14) into equation (12) we see that

$$C_\pm(\alpha, m) = \frac{\left[\left(L_m^2(-|\alpha|^2) - \mathrm{e}^{-4|\alpha|^2} L_m^2(|\alpha|^2)\right)\left(1 - \mathrm{e}^{-4|\alpha|^2}\right)\right]^{1/2}}{L_m(-|\alpha|^2) \pm \mathrm{e}^{-4|\alpha|^2} L_m(|\alpha|^2)}, \tag{15}$$

which is another expression different from equations (23) and (24) in [1]. In particular, when $m = 0$, equation (15) becomes

$$C_+(\alpha, 0) = \frac{1 - \mathrm{e}^{-4|\alpha|^2}}{1 + \mathrm{e}^{-4|\alpha|^2}}, C_-(\alpha, 0) = 1. \tag{16}$$

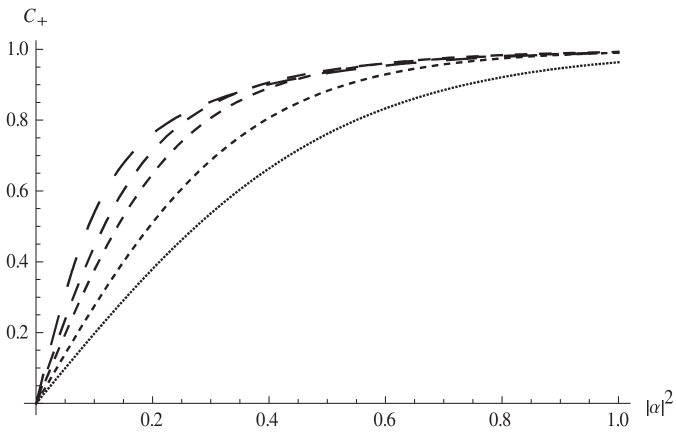

Figure 1. Concurrence of entanglement of $|\Psi_+(\alpha, m)\rangle$ as a function of $|\alpha|^2$ for the different photon excitations with $m = 0$ (solid line), $m = 1$(dashed line), $m = 3$ (dotted line), $m = 5$ (dash-dotted line) and $m = 20$ (dash–dash-dotted line), respectively.

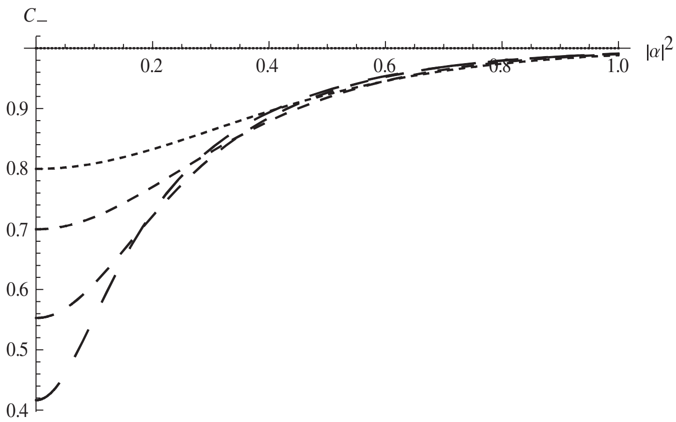

Figure 2. Concurrence of entanglement of $|\Psi_-(\alpha, m)\rangle$ as a function of $|\alpha|^2$ for the different photon excitations with $m = 0$ (solid line), $m = 3$ (dashed line), $m = 5$ (dotted line), $m = 10$ (dash-dotted line) and $m = 20$ (dash–dash-dotted line), respectively.

It implies that the concurrence $C_+(\alpha, 0)$ of ECS $|\Psi_+(\alpha, 0)\rangle$ increases with the values of $|\alpha|^2$, while $C_-(\alpha, 0)$ is independent of $|\alpha|^2$and $|\Psi_-(\alpha, 0)\rangle$ is a maximally entangled state.

In order to see clearly the influence of the concurrence with parameter m, the concurrences C for the state $|\Psi_\pm(\alpha, m)\rangle$ as a function of $|\alpha|^2$ are shown in figures 1 and 2. It is shown that $C_\pm(\alpha, m)$ increases with the increase of $|\alpha|^2$ for the given parameter m. Especially, the concurrence $C_\pm(\alpha, m)$ tends to unit for the larger $|\alpha|^2$. These conclusions are completely different from those of [1].

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